Source depth for solar p-modes

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ABSTRACT

Theoretically calculated power spectra are compares with observed solar p-mode velocity power spectra over a range of mode degree and frequency. The depth for the sources responsible for exciting p-modes of frequency 2.0 mHz is determined from the asymmetry of their power spectra and found to be about 800 km below the photosphere for quadrupole sources and 150 km if sources are dipole. The source depth for high frequency oscillations of frequency greater than $\sim$ 6 mHz is 180 (50) km for quadrupole (dipole) sources.

Subject headings: Sun: oscillations; convection; turbulence

1. Introduction

During the last ten years there has been increasing evidence that solar p-modes are excited as a result of sound generated by turbulent convection (e.g., Goldreich et al. 1994; Georgobiani et al. 2000). Acoustic oscillations with frequency above ∼5.5 mHz, the acoustic cutoff frequency in the solar atmosphere, provide good support for the general validity of this mechanism (Kumar 1994). The observed asymmetry of p-mode line profiles and the opposite sense of asymmetry in the velocity and the intensity power spectra have also been successfully modeled within the frame work of the stochastic excitation theory (Abrams & Kumar 1996; Roxburgh & Vorontsov 1997; Nigam et al. 1998; Kumar & Basu 1999a).

Mode excitation appears to be concentrated very close to the top of the convection zone (cf. Kumar 1994, Kumar & Basu 1999b), which is consistent with the theory of convective excitation. According to the mixing length theory of convection the location for the excitation of higher frequency acoustic waves should be a little higher up in the convection zone than lower frequency oscillations since the convective frequency increases with radius.

The theoretical modeling of line-asymmetry of low frequency p-modes provides a means to determine the depth at which these oscillations are excited. The source depth for high frequency oscillations ($\nu > 5.5$ mHz, the acoustic cutoff in the solar atmosphere) can be obtained from the frequency separation between adjacent peaks in the power spectrum. The purpose of this paper is to determine the source depth for both high and low frequency acoustic oscillations and to compare it with the depth expected from convective theory. This, we hope, will teach us about convective velocity etc. in the top few scale heights of the convection zone where the mixing length formalism is known to be very inaccurate.

2. Theoretical calculation of spectrum

The calculation of power spectra is carried out using the method described in Abrams & Kumar (1996) and Kumar (1994). Briefly, a coupled set of linearized mass, momentum and entropy equations, with a source term, is solved using the Green's function method. The source is parameterized by two numbers – the depth where the source peaks and the radial extent (the radial profile is taken to be a Gaussian function). We calculate power spectra for dipole and quadrupole sources using

$$P(\omega) = \left| \int dr S(r,\omega)\, \frac{d^n G_\omega}{dr^n} \right|^2 , \tag{1}$$

where $n = 0$ for dipole and 1 for quadrupole sources, and G_ω is the Green's function for the linearized set of non-adiabatic wave equations. Physically, dipole sources produce acoustic waves by applying a time dependent force on the fluid. Only fluctuating internal stresses are associated with quadrupole sources which have no associated net momentum flux.

The power spectrum of p-modes below the acoustic cut-off frequency of ~ 5.5mHz is distinctly asymmetric and the magnitude of asymmetry is a function of source depth which has been used by Abrams & Kumar (1996) and Kumar & Basu (1999b) to determine the source depth for low frequency modes. The frequency spacing between adjacent peaks above the acoustic cutoff frequency can be used to determine the source depth for high frequency waves (Kumar, 1994). These works provided preliminary evidence for frequency dependence of source depth. In this paper we carry out the fit to the best available observed low and high frequency solar spectra using an up to date. solar model, to refine the source depth determination as a function of wave frequency.

We use two solar models to calculate the theoretical power spectrum. One ("the standard model") is a standard model of the present Sun. It is constructed with OPAL opacities (Iglesias & Rogers 1996) supplemented by low temperature opacities of Kurucz(1991), and the OPAL equation of state (Rogers, Swenson & Iglesias 1996). Convective flux is calculated using the formulation of Canuto & Mazzitelli (1991), and the photospheric structure is calculated using the empirical $T - \tau$ relation of Vernazza et al. (1981). To check dependencies on different ways of calculating convective transport we have also used a similar model which uses the standard mixing length formalism to calculate convective flux. The second model ("the old model") is the old Christensen-Dalsgaard model used in Kumar (1994) to determine source depth for high frequency oscillation, and we use this model to determine the error in source depth resulting from the use of different solar models.

The observed power spectra used for the low frequency modes are the 360 day data from the Michelson Doppler Imager (MDI) on board SoHO (Solar and Heliospheric Observatory); low frequency modes have small line-width and require long time series to resolve their line profile. For the high frequency waves we used data from GONG obtained during months 10 to 22 of its operation. This period corresponds to a rough minimum in solar activity. The monthly spectra were averaged together to reduce noise.

3. The fitting process

The fit to the high-frequency part of the power spectra contains four parameters: the source depth, a constant background, an over all amplitude normalization factor (we normalize the amplitudes to 1 at 6.5 mHz) and a uniform linear frequency shift of the power spectra which is needed because the solar model we use is not perfect i.e., there is a frequency difference between the solar model and the Sun. Figure 1 illustrates the frequency difference between the standard model and the Sun. Since the source depth determines the inter-peak frequency spacings rather than the absolute frequency position of the peaks, this shift should not affect the source depth determination. The theoretically calculated power-spectra include the effect of ℓ–leakage and Nyquist folding (the Nyquist frequency for GONG data set is 8.333 mHz) before comparison with the observed spectra is made.

At low frequencies, we have the same number of free-parameters as Kumar & Basu (1999b), i.e. the peak amplitude (which is normalized to unity), the line width, and the background (which is taken to be constant over the frequency range of the spectrum we model). Unlike Kumar & Basu (1999b), we take into account possible distortions of the power-spectra due to m-leakage. This is clearly seen in very low frequency modes which have line-widths of the order of the spacing between modes of $\delta m = \pm 1$. The ℓ-leakage is also important and is included in our theoretically calculated spectra. The ℓ-leakage is estimated from the ratio of different ℓ-peaks at low frequencies of the observed spectrum; modes of degree $\ell \pm 2$ contribute about 15% of their power to the spectra for modes of degree ℓ whereas the leakage from $\ell \pm 1$ is $\sim 50\%$.

The best-fit source depth is determined by minimizing the merit function (cf. Anderson, Duvall & Jeffries 1990)

$$F_m = \frac{1}{N} \sum_{i=1}^{N} \left(\frac{O_i - M_i}{M_i} \right)^2 \tag{2}$$

where, the summation is over all N data points, O_i is the observed power and M_i the theoretically calculated power.

4. Results

A summary of the source-depths obtained with the standard solar model for different cases is shown in Table 1.

Figure 2 shows the quadrupole and dipole fits to the high frequency spectrum. The dipole sources seem to provide as good a fit to the observed spectrum, however they have a serious drawback described below.

In order to fit the observed spectrum, the theoretical power spectrum for the dipole sources, placed at a depth of 30–130 km for the best fit, has to be shifted by -61μHz. The spectrum calculated with quadrupole sources is shifted by a lower amount of -24μHz in order to match the observed high frequency peaks. The theoretically expected shift is about -11 μHz at 4.3 mHz due to the inaccuracy of the solar model we use. The difference between the theoretical and the observed p-mode frequencies increases rapidly with frequency (see fig. 1) and a shift of $\sim 20\mu$Hz at about 6 mHz is perhaps not unreasonable.

However, a shift of 61μHz, needed for dipole excitation, is about half the frequency spacing between peaks, and is much larger than can be accounted for as resulting from solar model error. No location for dipole sources provides an acceptable fit to the high frequency spectrum without a large frequency shift of the power spectrum and we therefore conclude that the wave excitation for high frequency waves in the Sun is quadrupolar.

The quadrupole excitation requires a source depth of between 60–250 km (the uncertainty being mainly a result of error in estimating the ℓ-leakage and background removal); the depth is measured from the top of the convection zone. The old model gives similar source depths, between 50 to 300 km for quadrupole source and 10 to 80 km for dipole sources. The use of different convective theories to calculate the mean solar model, such as the mixing length theory and the model of Canuto & Mazzitelli (1991), makes almost no difference to the determination of source depth given the uncertainty in the ℓ-leakage.

The source-depth for the high-frequency waves is smaller than what Kumar & Basu (1999b) found for low frequency p-modes of $\ell = 35$ using the line asymmetry. We repeat their work for low frequency p-modes of ℓ =55 & 60.

Figure 3 shows the theoretical power spectra for quadrupole and dipole sources superposed on the observed spectrum of a mode with $\ell = 60$, radial order $n = 4$, and frequency $\nu = 2.01$mHz. The figure of merit per degree of freedom for the best fit is 0.085 for quadrupolar as well as dipole sources. Unlike the high frequency case, the source depth required is quite different for the two source types. The quadrupole sources have to be at depths of 700—1000 km to match the observed line asymmetry, while the dipole sources have to be placed at a depth of 120—350 km. These results are consistent with those found by Kumar and Basu (1999) for modes of $\ell = 35$. The results for $\ell = 55$ & 60 are identical. The standard model indicates that the depth at which a 2 mHz mode starts to propagate is ~ 1500 km. Therefore, the excitation of the mode is taking place in the evanescent region.

The main result of this work is that the source depth for low frequency p-modes (2 mHz) is much larger than the source location for acoustic waves of frequency larger than ~ 5.5 mHz.

5. Discussion

The depth dependence of energy input rate to a p-mode due to quadrupole excitation is given by (Goldreich et al. 1994)

$$\frac{d\dot{E}}{dr} \sim \frac{2\pi\omega_\alpha^2}{5} r^2 \rho^2 v_\Lambda^3 \left|\frac{\partial \xi_\alpha^r}{\partial r}\right|^2 \frac{\Lambda^4 \left(\mathcal{R}^2+1\right) \mathcal{S}^2}{1+(\omega_\alpha \tau_\Lambda/\eta)^{15/2}}, \tag{3}$$

where ω_α is mode frequency, Λ is vertical coherence length of largest eddies which in the mixing length theory is a constant multiple of pressure scale height, v_Λ is the convective velocity, $\tau_\Lambda = \Lambda/v_\Lambda$ is the eddy turnover time, ξ_α^r is the normalized radial displacement eigenfunction for the mode, η is a constant factor of order $\pi/2$, $\mathcal{R}$ is the dimensionless compressibility of the mode which is close to unity for high order p-modes, and $\mathcal{S}$ is the horizontal coherence length of eddies and is taken to be same for all inertial range eddies.

Figure 4 shows this function for modes of frequency 1.7, 2.1 and 5 mHz for a standard solar model and the mixing length theory of convection. Note that the depth for 1.7 mHz mode is about 400 km below the photosphere whereas the peak excitation depth for 5 mHz waves is $\sim$ 50 km. Moreover, these depths are much smaller than what we obtained in the last section for the preferred quadrupole sources by the modeling of line asymmetry for low frequency modes and the frequency spacing between adjacent peaks in power spectra at high frequencies. This indicates some error in the mixing length model of convective in the top few scale heights.

A possible ad hock modification of the convective profile to yield larger source depth is to decrease the convective timescale τ_Λ in the top few scale heights of the convection zone. Since τ_Λ enters as a very high power in equation (3) most of the excitation is concentrated at the resonance layer, i.e. where $\tau_\Lambda \sim \omega_\alpha^{-1}$, and a modest decrease in τ_Λ is sufficient to reconcile the "observed" and the theoretical source depths. For instance to shift the quadrupolar excitation depth for 1.7mHz mode to 1000 km below the photosphere we need to decrease τ_Λ by a factor of about 2 in the top 1000 km of the convection zone; this shifts the peak excitation depth for 2.0 mHz mode to 900 km and the high frequency acoustic waves to $\sim$ 150 km. A decrease in τ_Λ by a factor of 1.5 causes most of the excitation for 2.0 mHz mode to occur at a depth of $\sim$ 700 km. A decrease in τ_Λ could be caused by an increase in the convective velocity or a decrease in the radial coherence length for turbulent eddies or a combination of the two. The numerical simulation of solar convection (Abbett et al. 1997) indicates that the convective velocity in the top 2000 km is larger than expected from MLT by a factor of $\sim$ 1.5. We should not expect a modification to τ_Λ to be a constant factor independent of r. In fact the "observed" depth for low and high frequency waves does not seem to be compatible with a constant, depth independent, decrease to τ_Λ, but instead suggests that the modification factor is larger at smaller depths.

We expect realistic numerical simulations of the solar convection zone, such as those carried out by groups in Copenhagen, Michigan and Yale, to provide a deeper understanding for the deficiencies in the convective modeling of the superadiabatic layer near the top of the solar convection zone and shed light on the underlying physical cause for the greater source depth that we find.

This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory. This work also utilizes data from the Solar Oscillations Investigation / Michelson Doppler Imager (SOI/MDI) on the Solar and Heliospheric Observatory (SOHO). SOHO is a project of international cooperation between ESA and NASA. This work is supported by a NASA grant NAG 5-8328.

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Table 1: Source depths

Frequency Range	Source depth (km) Quadrupole	Dipole
$\ell = 60$, $\nu > 6000$ mHz	60 – 250	30 – 130
$\ell = 60$, $\nu = 2.01$ mHz	700 – 1000	120 – 350
$\ell = 55$, $\nu = 1.72$ mHz	700 – 1000	120 – 350

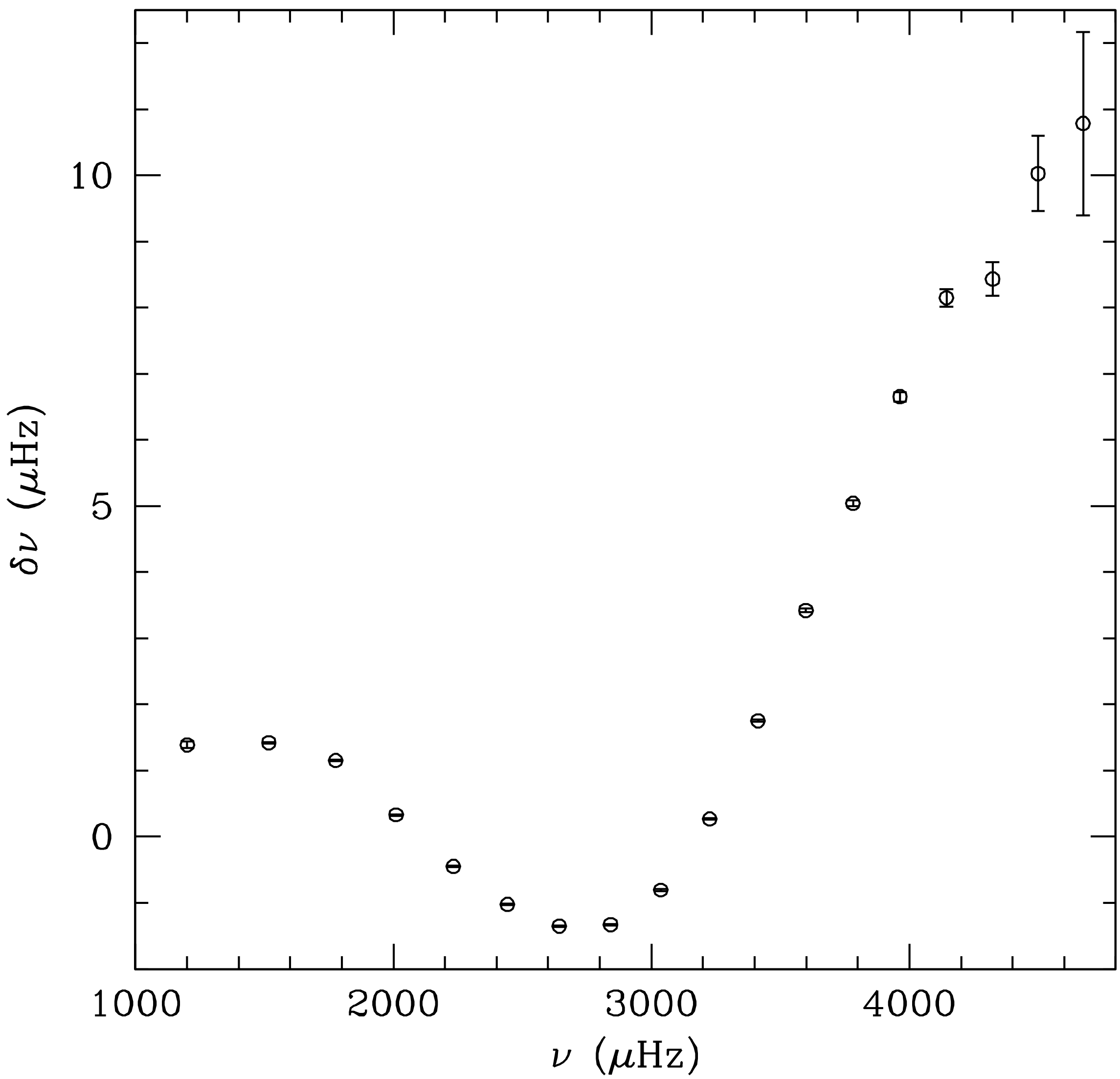


Fig. 1.— The difference between the adiabatic frequencies of the standard solar model and the observed solar p-mode frequencies. Note that the frequency difference increases rapidly at high frequencies.

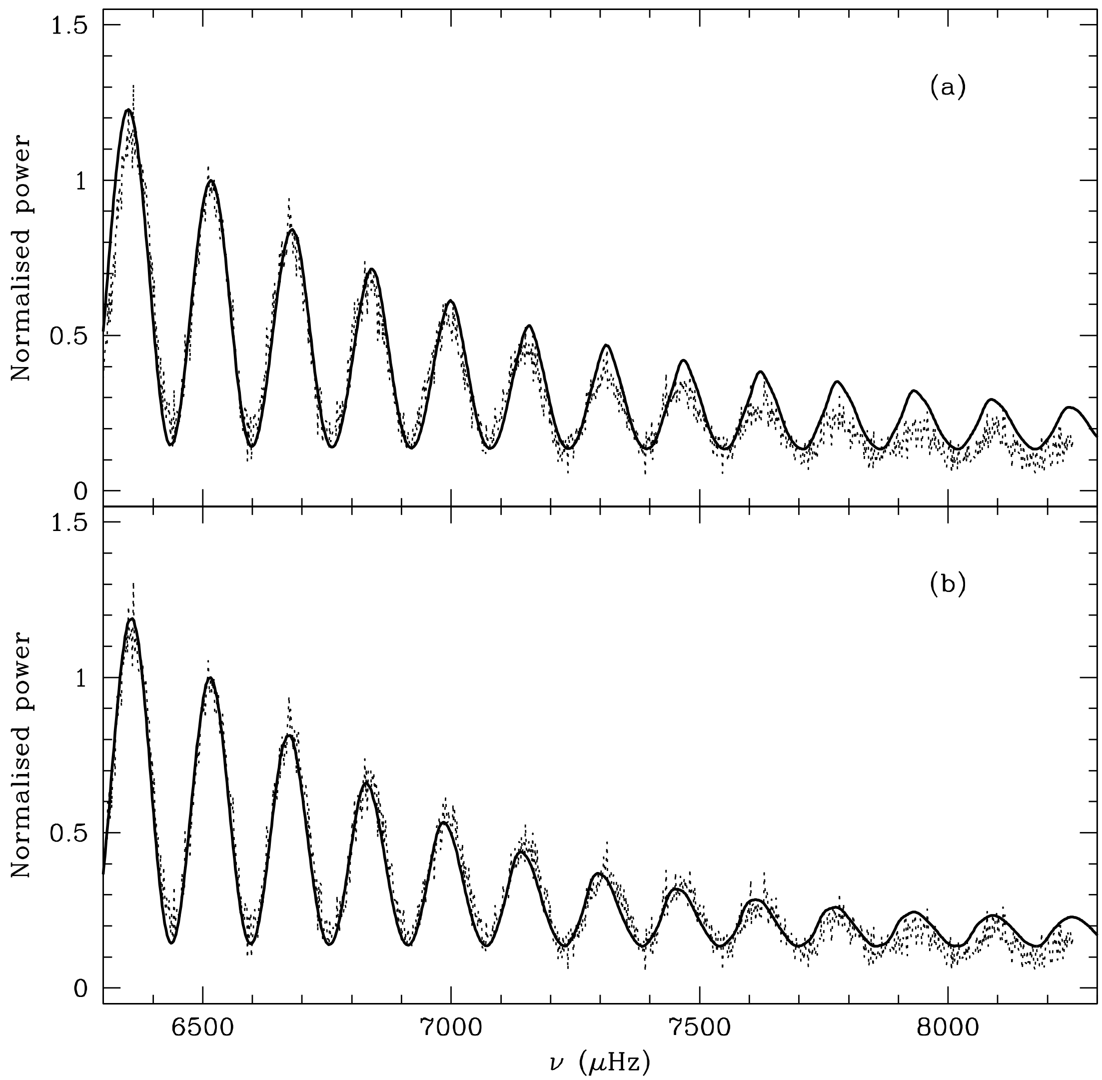


Fig. 2.— Panel (a): The best-fit quadrupole-source power-spectrum (continuous curve) superposed on the observed spectrum for $\ell = 60$ (dotted curve). The source depth of the theoretical curve is 189 km and the frequency shift required is -24 μHz. Panel (b): The best-fit dipole-source power-spectrum (continuous curve) superposed on the observed spectrum for $\ell = 60$ (dotted curve). The source depth of the theoretical curve is 40 km and the theoretical spectrum was shifted by -61 μHz in order to fit the observed peaks. The radial extent of sources is ~ 50 km.

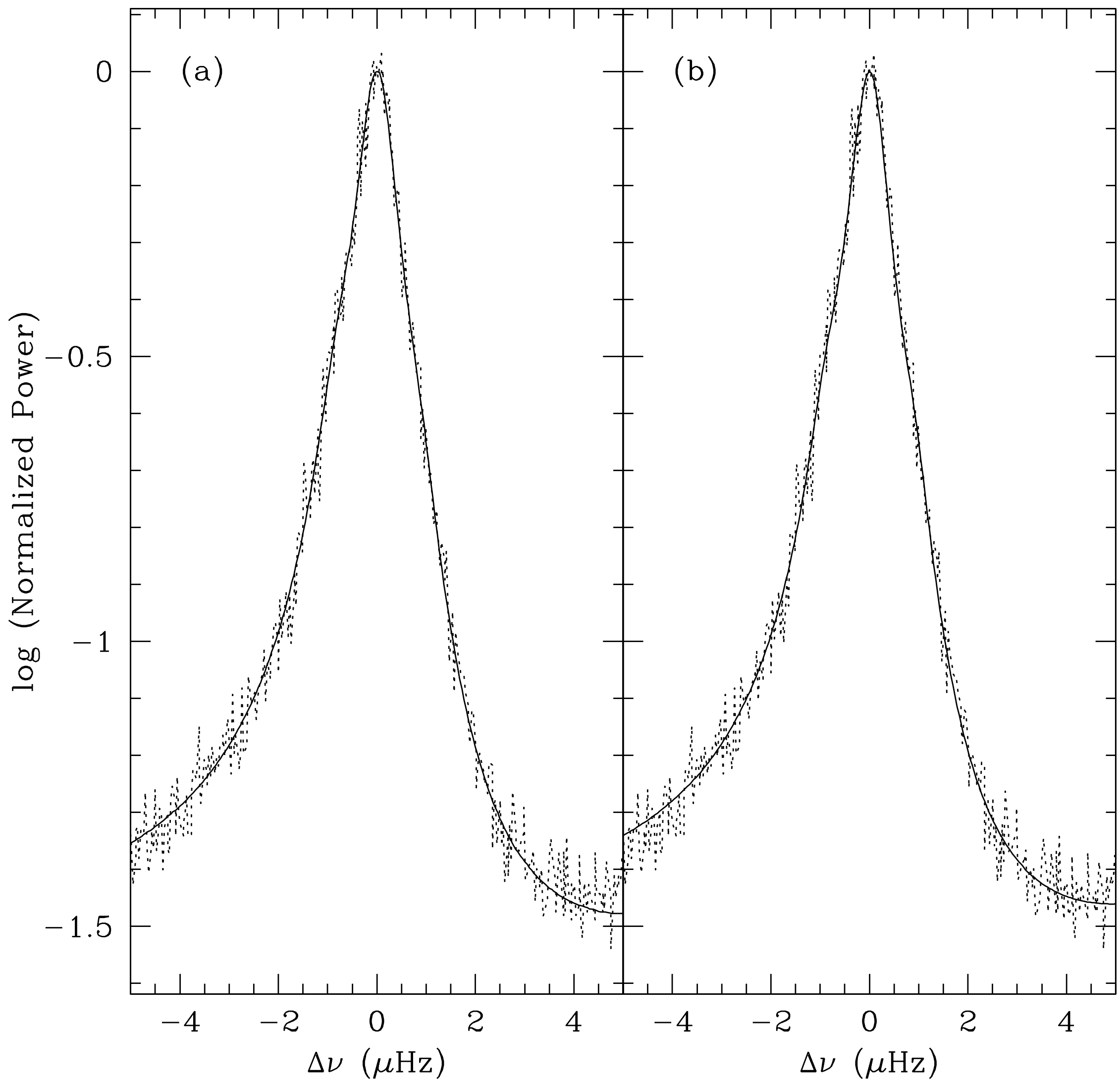


Fig. 3.— Panel (a): Quadrupole-source power spectrum for an $\ell = 60$ $\nu = 2.01$mHz mode. The source depth of the theoretical curve (continuous line) is 900 km. The dotted lines is the observed power spectrum. Panel (b): The fit for dipole sources at a depth of 100 km. The radial extent of sources is ~ 100 Km.

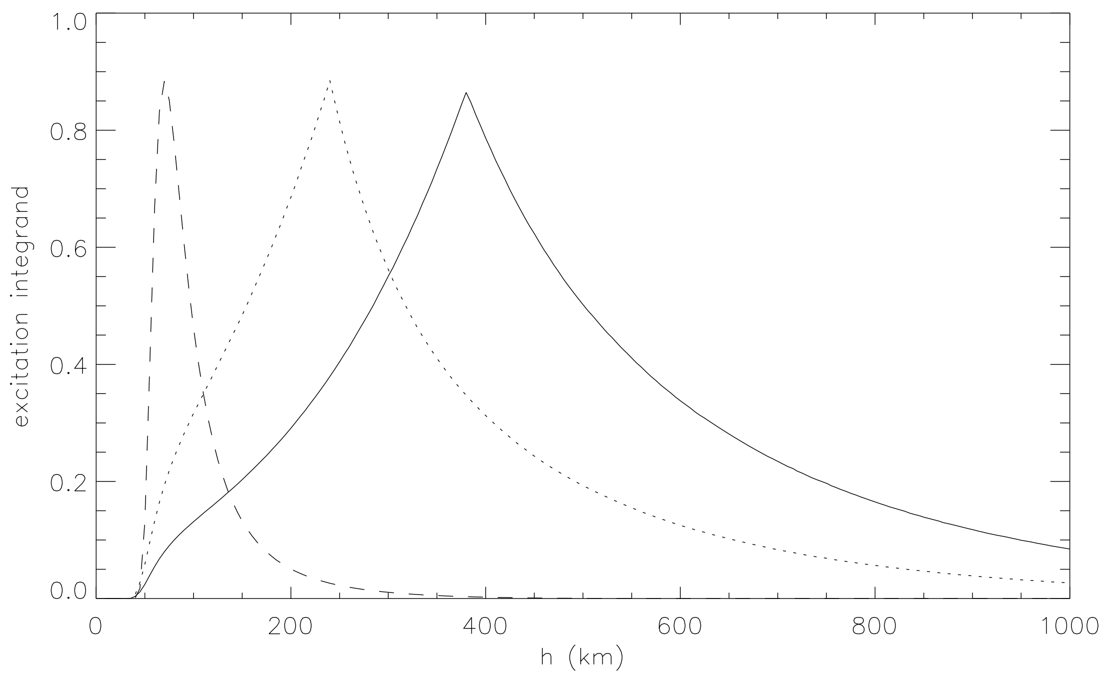
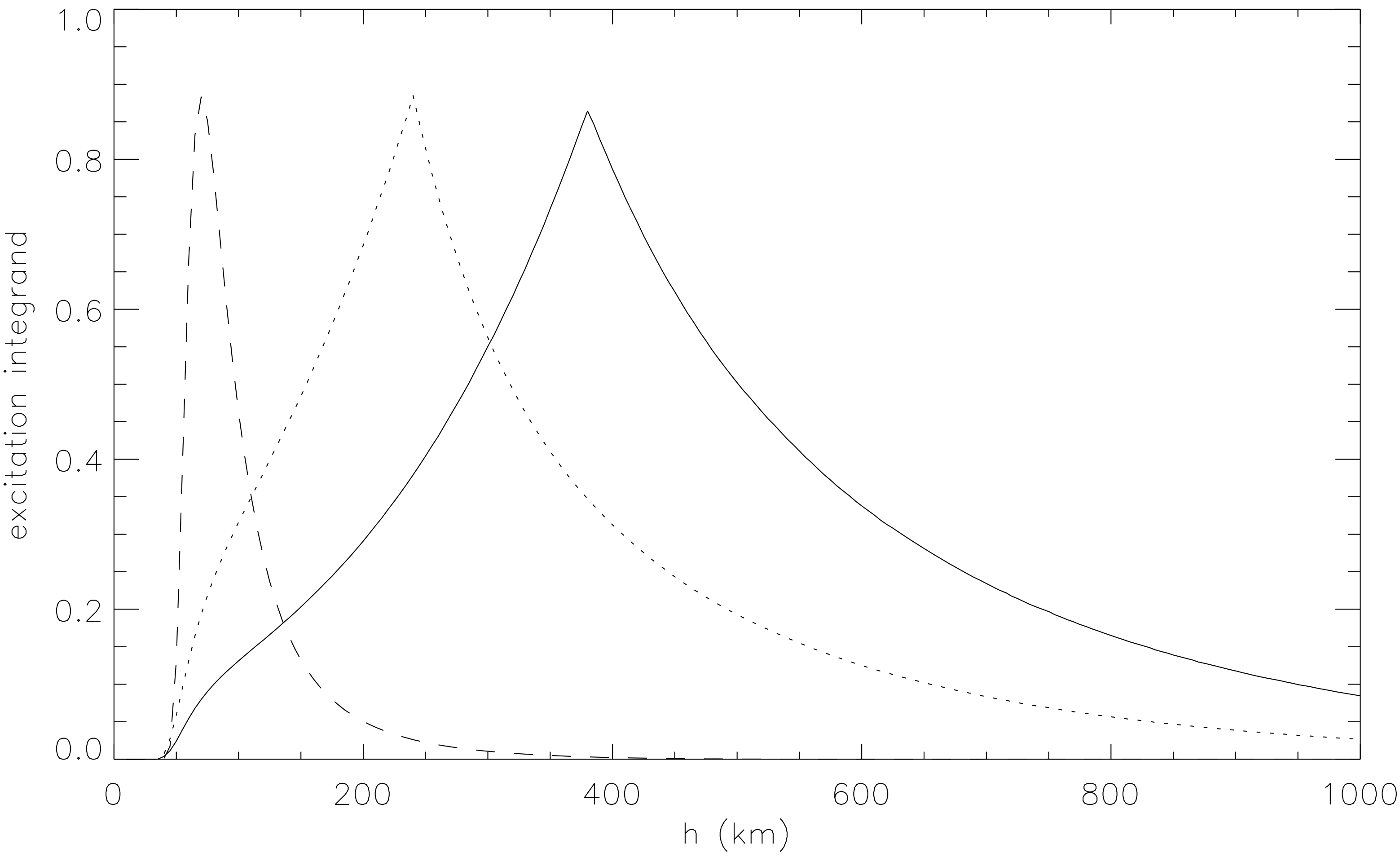


Fig. 4.— The rate of energy input in p-modes as a function of depth for quadrupolar sources (expression in eq. 3) is shown for three p-modes of frequency 1.7 mHz (continuous line), 2.1 (dotted curve) and 5.0 mHz (dashed line) all of degree $\ell = 55$. The convective velocity in the solar model was calculated using the standard mixing length theory.



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